Occupation fluctuation noise: A fundamental source of linewidth broadening in semiconductor lasers

Kerry Vahala and Amnon Yariv
California Institute of Technology, 128-95, Pasadena, California 91125

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In this letter we consider the effect of fast thermal fluctuations of electronic state occupancy on the field spectrum of semiconductor lasers and derive for the first time an expression for the resulting power independent linewidth contribution. The magnitude and temperature dependence of this linewidth component agree reasonably well with measurements of a power independent linewidth made by Welford and Mooradian.

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Although the ultimate spectral purity of lasers is set by quantum effects, ambient fluctuations, such as room microphonics, normally outweigh those of quantum origin giving lasers a field spectrum linewidth that is essentially power independent. Semiconductor laser systems are an exception. Due to their extremely high gain and strong amplitude-phase coupling, their noise properties are primarily determined by quantum effects. In particular, measurements have verified that the field spectrum linewidth of semiconductor lasers varies inversely with output power in accordance with the modified Schawlow–Townes linewidth formula[1–3]:

$$\Delta\nu = \frac{\hbar\nu E_{cv} v_g \ln(1/R)}{4PL}(1+\alpha^2), \tag{1}$$

where ν is the lasing frequency, E_{cv} is the spontaneous emission rate, v_g is the group velocity, R is facet reflectivity, P is output power per facet, L is the cavity length, and α is the linewidth broadening factor due to amplitude-phase coupling. This inverse power broadening results from spontaneous emission into the lasing mode. Of course, Eq. (1) is useful for predicting actual linewidths only below a certain output power level, since with increasing power other mechanisms of quantum (or ambient) broadening may eventually become dominant. Welford and Mooradian have observed such an additional linewidth component.[4] This linewidth is independent of output power and increases with decreasing temperature. After eliminating mechanisms, such as pump fluctuations, as causes of the observed broadening, they proposed electron number fluctuations in the active region and the resulting fluctuation of refractive index as a possible broadening mechanism. By assuming a number fluctuation of the form $\langle \Delta N^2 \rangle = N$ they derived an rms frequency deviation of the form:

$$\Delta\nu_{\rm rms} = \frac{\Gamma\nu}{\mu}\left(\frac{n}{V_c}\right)^{1/2}\frac{d\mu}{dn}, \tag{2}$$

where Γ is the optical confinement factor, μ is the refractive index, n is the carrier density, and V_c is the volume occupied by the carriers. Using values for $d\mu/dn$ which they inferred from frequency shift measurements below lasing threshold, the calculated values for $\Delta\nu_{\rm rms}$ were in agreement with their linewidth data.

Expression (2), however, is the rms frequency deviation of the resonant frequency of a passive Fabry–Perot etalon and not the spectral width of the laser field. These quantities are quite different. Calculation of linewidth requires, in addition to the parameters appearing in (2), a characteristic relaxation time for the fluctuation under consideration (i.e., carrier number fluctuation). We also have doubts that carrier number fluctuations explain the observed residual (high power) linewidth. In an analysis of semiconductor laser noise which includes the carrier density as a dynamic variable and the carrier density dependence of the refractive index it was found that carrier density (or carrier number) fluctuations do not contribute to the field spectrum linewidth.[5] This is due essentially to gain clamping. In this letter we will show that another mechanism, not yet considered, can cause a power independent linewidth contribution with the correct temperature dependence. This mechanism is the fluctuation of state occupancy in the conduction and valence bands caused by intraband thermalization or in short "occupation fluctuation noise." These fluctuations occur even when the total carrier number is fixed. To our knowledge, this is the first time these fluctuations have been included in a calculation of semiconductor laser linewidth.

A common way to specify the state of a semiconductor laser (i.e., its operating point) is to give the carrier density n and photon density p. Implicit in such a description is that state occupation in the conduction and valence bands is determined by appropriate quasi-Fermi distribution functions. Otherwise, parameters, such as gain and refractive index, would not be uniquely specified by the particular choice of n and p. If the occupation of states is described by a quasi-Fermi distribution, then choice of a time scale large compared to the intraband thermalization time (< 1 ps) is also implied by this description. Now consider a fluctuation of carrier density away from steady state. The macroscopic variable n returns to its steady state value via relaxation oscillations with a characteristic time on the order of nanoseconds. At the microscopic level of the individual electronic state the time average occupancy of the qth electronic state [i.e., $\bar{\rho}_q(t)$] oscillates as shown in part (a) of Fig. 1. In fact, since state occupancy is linked to carrier density through the quasi-Fermi level, all state occupancies oscillate in unison to return n to its steady state value. These oscillations, however, represent state occupancies which are time averaged over many intraband thermalization times. While the slow

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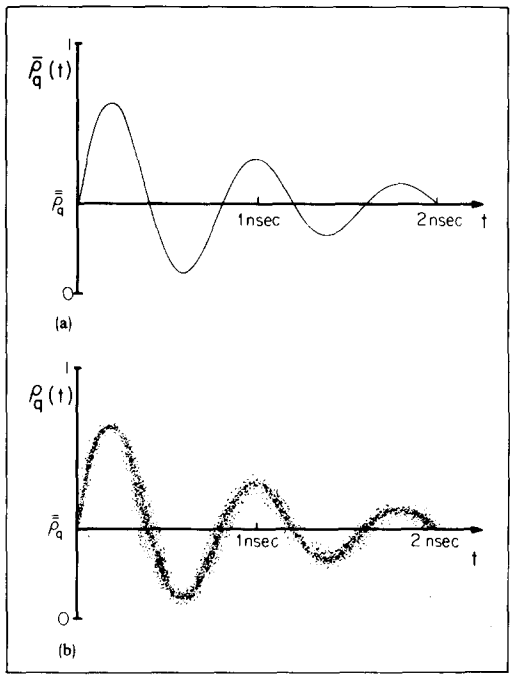

FIG. 1. Part (a) Response of time-averaged state occupancy when inversion is perturbed from steady state. Averaging is done over several intraband relaxation times. Part (b) Actual response (i.e., no time averaging) showing the fast thermal fluctuations of state occupancy.

relaxation oscillation occurs complex forces keep the population in quasi-thermal equilibrium and this thermalization causes fast fluctuations of state occupancy. Thus, on a time scale comparable to the intraband thermalization time the state occupancy exhibits fluctuations like those shown in part (b) of Fig. 1. Unlike the slow fluctuations in (a) [or the envelope in (b)] these fast thermal fluctuations are uncorrelated between states. Consequently, any macroscopic description of noise (i.e., one which considers fluctuations of carrier density or carrier number) necessarily neglects this class of fluctuations. To calculate their effect on the field spectrum linewidth requires the analysis to begin at the microscopic level of the electronic state.

The approach we take will be to calculate the perturbation $\Delta\mu_T$ to refractive index μ caused by these fluctuations. This, in turn, will give the field phase $\varphi(t)$ and the field spectrum linewidth according to expressions given in Ref. 5:

$$\dot{\varphi} = -\frac{\Gamma\omega}{\mu}\Delta\mu_T, \tag{3}$$

$$\Delta\nu = \frac{1}{2\pi}\lim_{\tau\to\infty}\frac{\langle[\varphi(t+\tau)-\varphi(t)]^2\rangle}{\tau}, \tag{4}$$

where $\langle\ \rangle$ denotes ensemble averaging. $\Delta\mu_T$ will be calculated using the density matrix formalism under the assumption of rigorous **k** selection rules. We will also assume that on time scales of interest the states in each band are quasi-independent so that in essence our model is equivalent to an ensemble of collisionally broadened two-level systems with different transition energies. The contribution to refractive index at frequency ω caused by this ensemble is calculated in Ref. 6. We merely quote the result given there:

$$\Delta\mu(t) = \sum_q \frac{A}{2\mu}[\rho_{cq}(t)-\rho_{vq}(t)]\frac{\omega-\omega_q}{(\omega-\omega_q)^2+1/T_2^2}, \tag{5}$$

where A is proportional to the transition matrix element, $\rho_{cq}(\rho_{vq})$ is the occupancy of the qth conduction (valence) band state, ω_q is the transition frequency, and T_2 is the intraband scattering time due to phase destroying collisons. A and T_2 are taken as energy independent parameters. We note that Eq. (5) gives only the component of refractive index due to interband transitions. The component due to free-carrier absorption is usually an order of magnitude smaller and will be neglected in this analysis.[6] On a time scale comparable to the intraband relaxation time we assume the state occupancies relax according to the following simple relations[7]:

$$\dot{\rho}_{cq} = (\rho_{cq}-\bar{\rho}_{cq})/T_c, \tag{6}$$

$$\dot{\rho}_{vq} = (\rho_{vq}-\bar{\rho}_{vq})/T_v, \tag{7}$$

where T_c and T_v are the intraband relaxation times of the conduction and valence band occupancies (assumed to be energy independent for simplicity) and where $\bar{\rho}_{cq}$ and $\bar{\rho}_{vq}$ are the slowly varying conduction and valence band occupancies depicted in part (a) of Fig. 1. The time-averaged rate equations governing the slowly varying quantities $\bar{\rho}_{cq}$ and $\bar{\rho}_{vq}$ are not required for the purposes of this analysis. We now separate the fast fluctuation component of $\Delta\mu(t)$ by defining $a_q \equiv \rho_{cq}-\bar{\rho}_{cq}$ and $b_q \equiv \rho_{vq}-\bar{\rho}_{vq}$:

$$\Delta\mu_T(t) = \sum_q \frac{A}{2\mu}[a_q(t)-b_q(t)]\frac{\omega-\omega_q}{(\omega-\omega_q)^2+1/T_2^2}. \tag{8}$$

The assumptions involved in the derivation of (5) and (8) require that a_q and b_q fluctuate slowly in comparison to T_2^{-1} or equivalently that $T_c, T_v \gg T_2$. Since $T_c, T_v \approx 1$ ps whereas $T_2 \approx 0.1$–0.3 ps this condition is normally satisfied.[6]

Since the population in each band is in quasi-thermal equilibrium, the grand canonical ensemble can be applied to calculate the fluctuation moments $\langle a_q^2\rangle$ and $\langle b_q^2\rangle$[8]:

$$\langle a_q^2\rangle = \kappa T\frac{\partial\bar{\rho}_{cq}}{\partial\epsilon_c}, \tag{9}$$

$$\langle b_q^2\rangle = \kappa T\frac{\partial\bar{\rho}_{vq}}{\partial\epsilon_v}, \tag{10}$$

where κ is Boltzmann's constant, T is the absolute temperature, and $\epsilon_c(\epsilon_v)$ is the quasi-Fermi level for the conduction (valence) band. Strictly speaking, $\langle a_q^2\rangle$ and $\langle b_q^2\rangle$ are time dependent moments since $\bar{\rho}_{cq}$ and $\bar{\rho}_{vq}$ are time dependent. For typical fluctuations of carrier density, however, $\bar{\rho}_{cq}$ and $\bar{\rho}_{vq}$ are only weak perturbations of the true steady state $\bar{\bar{\rho}}_{cq}$ and $\bar{\bar{\rho}}_{vq}$ (the large fluctuation shown in Fig. 1 is highly exaggerated for illustrative purposes). For this reason we are justified in replacing $\bar{\rho}_{cq}$ and $\bar{\rho}_{vq}$ in (9) and (10) by their true steady state values. Assuming the system can be modeled as Markoffian, (6), (7), (9), and (10) yield[9]

$$\langle a_q(t+\tau)a_p(t)\rangle = \kappa T\frac{\partial\bar{\bar{\rho}}_{cq}}{\partial\epsilon_c}\exp\left(-\frac{\tau}{T_c}\right)\delta_{qp}, \tag{11}$$

$$\langle b_q(t+\tau)b_p(t)\rangle = \kappa T\frac{\partial\bar{\bar{\rho}}_{vq}}{\partial\epsilon_v}\exp\left(-\frac{\tau}{T_v}\right)\delta_{qp}, \tag{12}$$

where δ_{qp} is the Kronecker delta. (3), (8), (11), and (12) are used to form the argument of the limit in (4). Simplification

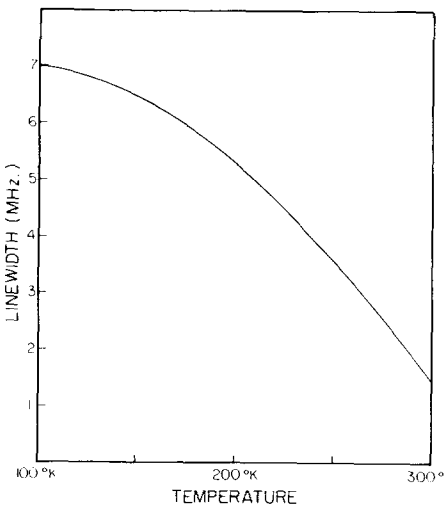

FIG. 2. Calculated power independent linewidth vs temperature.

proceeds as described in Ref. 5. The resulting laser linewidth is given by the following expression:

$$\Delta\nu = \frac{\kappa T}{V_c}\left(\frac{\pi\omega A\Gamma}{\mu^2}\right)^2 \int_{-\infty}^{+\infty} D(\Omega)\left(\frac{\omega-\Omega}{(\omega-\Omega)^2+1/T_2^2}\right)^2 \times\left(T_c\frac{\partial\bar{\rho}_c}{\partial\epsilon_c}+T_v\frac{\partial\bar{\rho}_v}{\partial\epsilon_v}\right)d\Omega, \tag{13}$$

where $D(\Omega)$ is the effective density of states. $\Delta\nu$ represents a contribution to field spectrum linewidth in addition to that resulting from spontaneous emission into the lasing mode [Eq. (1)]. Since the fluctuations a_q and b_q are regulated only by the thermal forces which maintain the quasi-Fermi distribution, $\Delta\nu$ is independent of the output power of the laser [as can be readily seen from Eq. (13)]. In Fig. 2 $\Delta\nu$, calculated from (13), is plotted versus temperature. The parabolic band effective density of states with effective masses characteristic of GaAs, and the Kane matrix element were used for the calculation.[6] ω was set at the peak of the gain spectrum which was calculated as shown in Ref. 6. The carrier density at each temperature was selected to maintain a constant gain spectrum peak corresponding to a photon lifetime of 2 ps. In addition we assumed $T_c \approx T_v = 1$ ps at 300 K (Ref. 6) and scaled these quantities linearly to a value of 7 ps at 100 K. The endpoint at 100 K is consistent with the temperature dependence of electron-phonon and electron-electron scattering times[10,11] (Note: impurity scattering is neglected in this analysis because it does not alter the energy of the scattered state.) Our selection of a linear scale is arbitrary. The following parameters were also used in the calculation: $\omega = 2.2\times10^{15}$ rad s^{-1}, $\Gamma = 0.4$, $\mu = 3.5$, $T_2 = 0.3$ ps,[6] and $V = 0.8\times10^{-10}$ cm^{-3}. The temperature dependence of T_2 was neglected in the calculation since $\Delta\nu$ was only weakly dependent upon it. These parameters are typical of the transverse junction stripe structure measured by Welford and Mooradian. The magnitude and temperature dependence of $\Delta\nu$ agree favorably with the experimental values which are 1.9, 5.2, and 8.4 MHz at 273, 195, and 77 K. Indeed, the agreement is surprisingly good since our model assumes a simplified band structure and transition rules.

In conclusion, we have proposed a new noise producing mechanism in semiconductor lasers. It is the fast fluctuations of electronic state occupancy which accompany intraband thermalization. We have also shown that this mechanism causes a power independent component of field spectrum linewidth. The magnitude and temperature dependence of this component are in reasonable agreement with the measured values of a power independent linewidth by Welford and Mooradian. We believe that this new mechanism could explain their data.

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